The mathematics of model flying: Slices of Pi (π)

Many people like me think that BBC television in the UK has lost its way. It contains the same repetitive ‘flogging a dead horse’ drivel about crime, houses, selling junk, cooking, ‘travel by celebrities’, game shows and gardening as the low-grade stuff on the commercial channels. By contrast its radio programmes continue their high quality. One example is the science programme ‘The Curious Cases of Rutherford and Fry’. Both Adam Rutherford and Hannah Fry are professors, Adam in the biological sciences and Hannah in maths. The one on BBC radio 4 on 8 September 2022 dealt with Pi. As we all know Pi is to do with circles, but they showed that it is much, much more.

In 1897 Edward G Goodwin persuaded the Indiana State Legislature to change the value of Pi to 3.2. They voted 67 to 0 in favour. He said that this ‘new mathematical truth’ (does that sound familiar) would make things simpler and as he had copyrighted it Indiana would benefit from royalties. Fortunately Professor Clarence A. Waldo educated the legislators that the accepted value of Pi is a fundamental fact of the universe and cannot be altered.

At its simplest Pi is the ratio of the circumference of a circle to its diameter. To sixteen significant figures it is 3.141592653589793. Why is that value so important? NASA uses it in the equations that guide its space craft, as explained by Mission Director Marc Rayman in the programme. The Voyager 1 spacecraft has so far reached 23 000 000 000 km away from earth and this level of accuracy means a potential error of about 6 millimetres in its path at that distance. If Goodwin’s value had been used a space craft would have missed even our nearby moon by several kilometres (2% of the moon’s diameter). The astonishing, perfect hit that the spacecraft made on the asteroid in September 2022 illustrates the accuracy of that value of Pi.

The digits of Pi never repeat. To be exact we have only found that to be true up to 62.8 million million places and that might not be enough of course. Pi is called an irrational number because it cannot be found by any **ratio** of two whole numbers. 22/7 is often used for Pi but it is approximate at 3.142857142857 with the last six digits then repeating. For most ordinary purposes it is fine being only four ten-thousandths out (0.0004). If you really need a more accurate ratio try 333/106 or355/113.

If you listen to the BBC programme you will learn some jaw dropping facts. The overall idea was that Pi is found everywhere and the programme described a wide range of examples. Its irrational value is due to inadequacy in our maths and number system. To my mind this does have some sense as our number system is linear whereas space is curved and curved lengths involve Pi. I have contacted the programme to suggest that, adding the fact that I might be talking through a less fluent orifice (copy at the end).

The ancient Greek, Archimedes, suggested a method to find Pi. He said take a polygon with all sides equal in length. Calculate the area of each segment. Then increase the number of sides until the length of each approaches zero. You then have a circle and can use the same method to find the area. Wiki ‘Area of a circle’ says: One method of deriving this formula, which originated with Archimedes, involves viewing the circle as the limit of a sequence of regular polygons. The area of a regular polygon is half its perimeter multiplied by the distance from its centre to its sides, and the corresponding formula – that the area is half the perimeter times the radius – namely, $A = 1/2 \times 2\pi r \times r$, holds in the limit for a circle.

I confess that I haven't yet tried the experiments in this article. They were tried for the programme. If you do try them please let me know the results.

Experiment one: You can measure Pi with a pendulum with a length of g/4, where g is 9.81. Each swing back and forth will be Pi seconds. If you measure ten or twenty swings to improve accuracy and then divide by ten or twenty you will be surprised at the answer. In the programme they hung up a real pie, in this case a chicken and mushroom one, and got the answer 3.141.

Remember this from school?

$$T = 2\pi\sqrt{\frac{L}{g}}$$

If L is a quarter of g then L/g = ¼
The square root of that is ½
T = 2Pi x ½ = Pi

Experiment two: to cannon two billiard balls. One should be near a cushion or other bouncy surface. You hit the other into it straight using a cue. Count the collisions including that with the cushion. You will find it to be three as the cushion ball bounces back from the one you hit. If you now use a heavier ball for the one you hit with the cue you will get an increasing number of collisions. Divide by the mass ratio and you will get more significant figures of Pi. For example if the moving ball is ten times the stationary one you should get 31 bounces which divide to an answer 3.1. Not sure how you will count them. Maybe slow motion video on your phone?

To illustrate the probable infinity of the number of places of Pi it was suggested that if we replaced two consecutive numbers with the letter in the alphabet, for example 04 being d, then somewhere in the sequence of digits will be every work of Shakespeare and of course your name and birthday. Unlike Shelley Berman's monkeys, it would not stop at 'To be or not to be, that is the gfornenplatz.'

Galileo said, 'If the universe was written in a book, the language it would be written in would be mathematics.'

Angles are best measured in radians. Any living or artificial intelligence would recognise it whereas our 360 degree system is arbitrary. Indeed in some European countries they divide the right angle into 100 degrees and call it centigrade. Years ago I got baffled looks in Italy when I asked the temperature in centigrade rather than celsius.

Experiment three: Are you a very poor darts player? In other words do you throw at random? If so put a round dart board inside a square that exactly fits it. Throw a large number of random darts. Most will go into the round board. Some will go into the small areas at each corner. Count the number in each area. You know the area of the square. The ratio of the number in the circle to the total number thrown will give the area of the circle compared with the square. You know the radius so can calculate Pi. If you are a decent darts player you could simulate incompetence by wearing a mask over your eyes, provided you protect the wall, and people nearby, against dart holes. You will need a system of verbal messages to avoid landing a dart in your assistant and a lot longer to play.

Another astounding fact and warning was that every person who ever works with Pi, dies. And not a scarab beetle nor mummy in sight.

Why is Pi important to us model flyers?

It is in every one of the equations that describe radiation. As our signals, whether light or radio frequency, travel out in straight lines the area they cover increases by a factor including Pi. This is implicit in the inverse square law that we ignore at our peril.

There is another and perhaps more compelling reason. Whatever you do never invite someone called ‘Pie’ to your house for dinner. There are only two pi in a pie so he’ll have half of it.

Sent by me to the programme on 9th September 2022 by email
Hello Hannah and Adam

I always love your programme but the one on Pi was outstanding. Thank you.

As it went on I had either an epiphany or a moment of mental drivel. It was when you talked about aliens and the universality of Pi. Could it be that Pi is irrational because our number system is linear, so is one-dimensional, but the universe is curved so three-dimensional? Could it explain why when we work with practical space we have to have Pi as a 'fudge factor' due to this mis-match?

By the way this might also explain the need for i or j - the square root of minus one - in science and engineering. This adds one more 'imaginary' dimension to the number system. Should you have a programme on root minus one?

In the end I got my moment of glory. On 16th February 2023 my name came up and we had a discussion about i. Not quite Andy Warhol’s idea of fiftenn minutes of fame but it’ll do.

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